Computer Algebra Systems Activity: Developing the Quadratic Formula

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Topic: Developing the Quadratic Formula

Notes to the Teacher:

a) This activity is designed to use the CAS on the TI-Nspire CAS calculator to enhance understanding and instruction. All screen shots are from the TI-Nspire CAS.

b) The instructions for the activity assume that the user has some elementary experience with a CAS. Novice users should complete the activity TI-Nspire CAS An Introduction before attempting this activity.

c) The activity is presented in a **Teacher Version**, with all screen shots and solutions present, as well as a **Student Version**, which can be duplicated and handed out to students.

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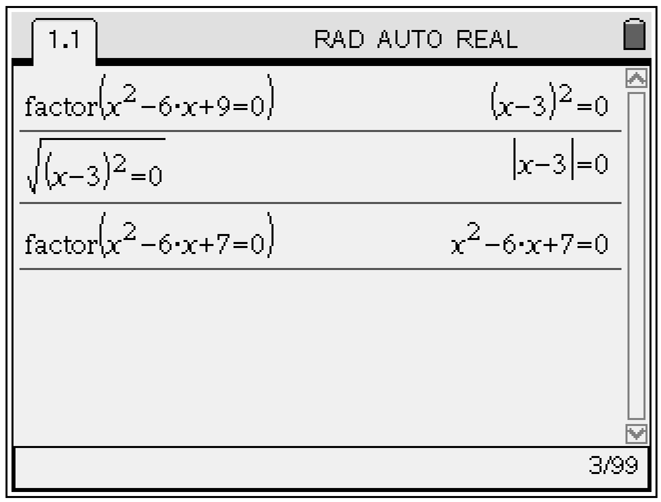
Teacher Version:

Note: This method is not intended as a substitute for developing the quadratic formula in the traditional manner.

1. Solve the quadratic equation $x^2 - 6x + 9 = 0$.

Press **menu**, select **3:Algebra** and then **2:Factor**.

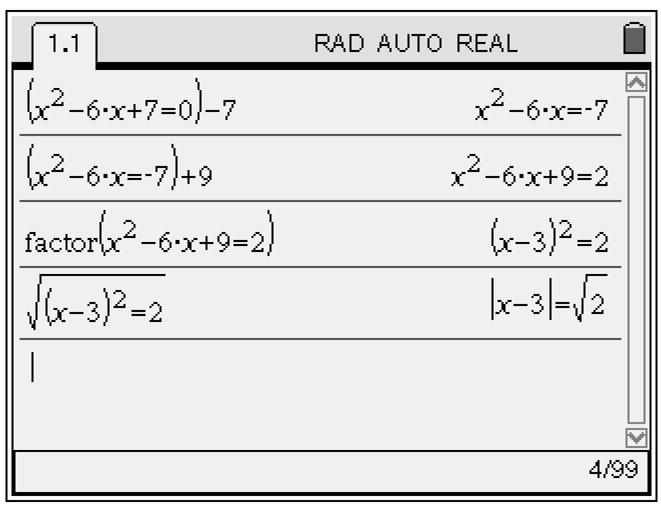
Type in the equation. Press **enter**.



Take the square root of both sides. **Tip**: You can minimize the typing required by using **ctrl** c and **ctrl** v to cut and paste.

Note that the only solution to the equation $|x - 3| = 0$ is $x = 3$.

2. Now consider the equation $x^2 - 6x + 7 = 0$. Inspection shows that this trinomial cannot be factored into two binomials using only integers. You can confirm this by attempting to factor the equation using the CAS.



Try another approach. Subtract 7 from both sides. Think back to step (1) above. The left side can be turned into a perfect square by adding 9. You may have seen this pattern before: to make a perfect square, take half of the coefficient of the middle term, and square it. This is called "completing the square". Add 9 to both sides.

Factor both sides, and then take the square root of both sides, following the pattern from step (1).

This time, there are two solutions to the absolute value equation:

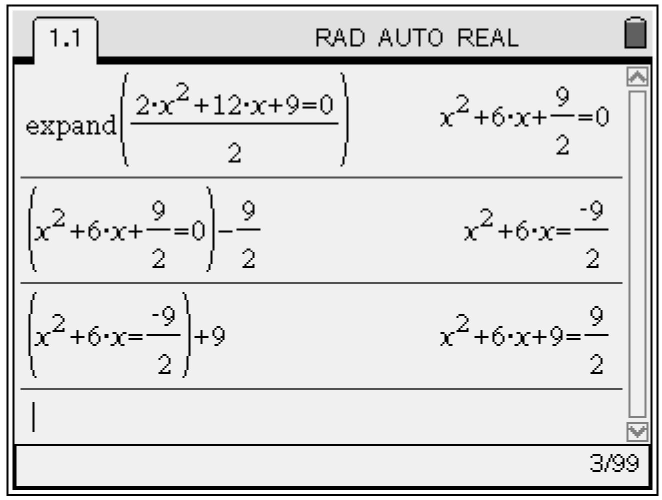
Either $x - 3 = \sqrt{2}$ or $x - 3 = -\sqrt{2}$.

Therefore, $x = 3 + \sqrt{2}$ or $x = 3 - \sqrt{2}$.

3. Consider the equation $2x^2 + 12x + 9 = 0$. You can easily confirm that this trinomial cannot be factored using integers. However, if you divide both sides by 2, you can follow a "complete the square" method, as in step (2). Divide both sides by 2.

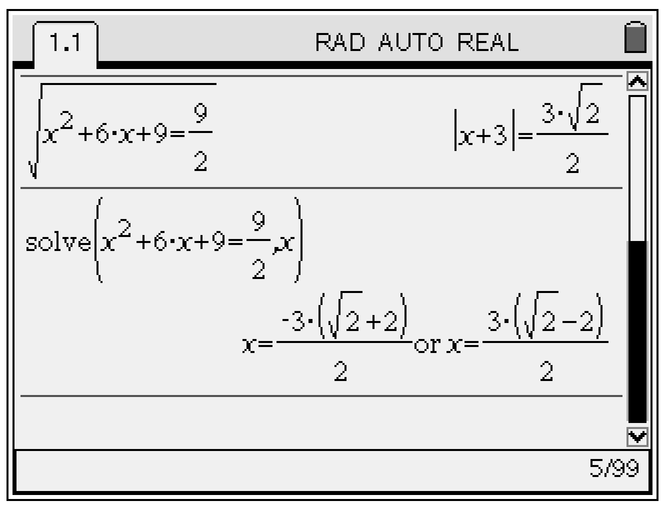
Use **3:Expand** from the **Algebra** menu to simplify the division. Then, subtract 9/2 from both sides.

Take half of the coefficient of the resulting *x* term, square it, and add to both sides.

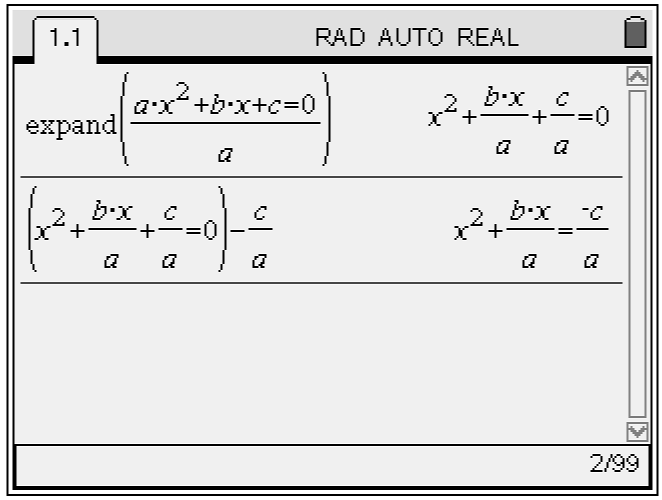


Take the square root of both sides.

The absolute value equation has two solutions. You can solve for *x* in the usual way, or you can use **1:Solve** under the **Algebra** menu.

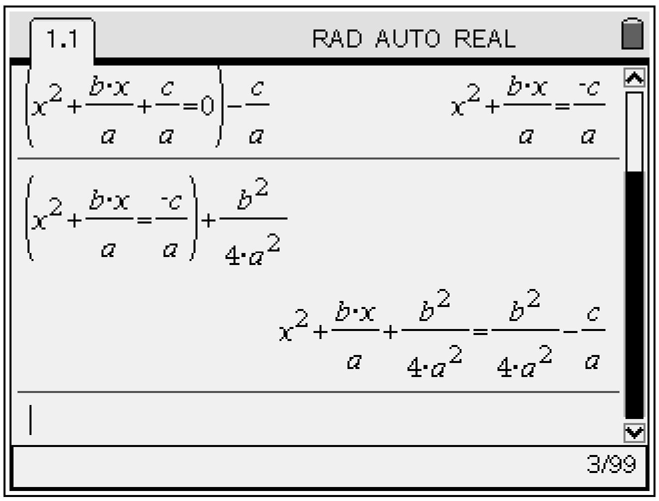


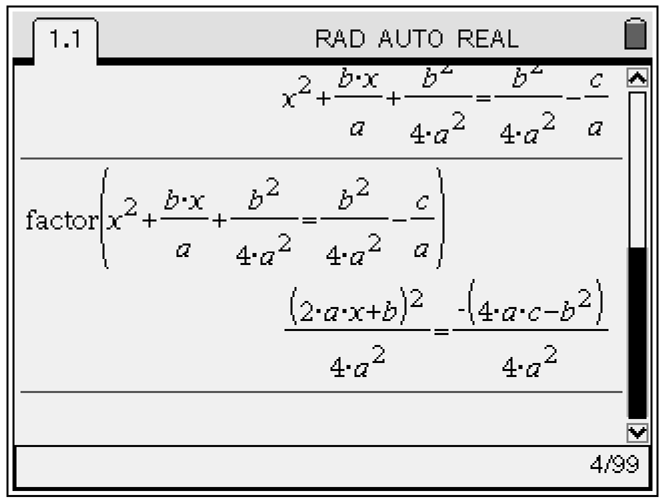
4. You will now use the power of CAS to develop a solution for the general quadratic equation, $ax^2 + bx + c = 0$. Follow steps similar to those in part (3) to solve for x.



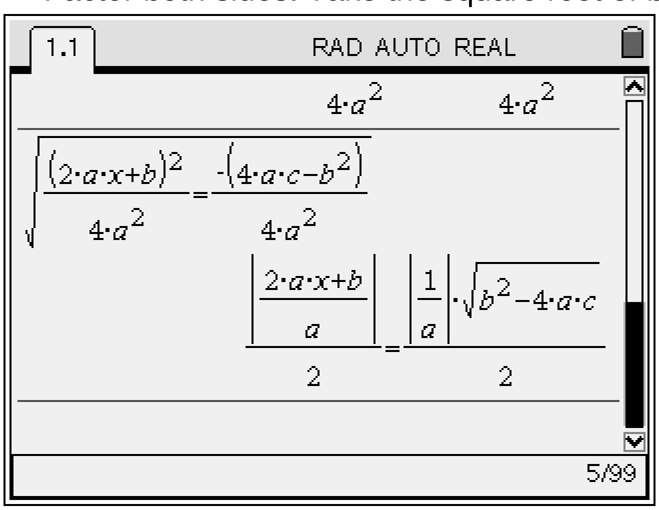
Divide both sides by a, and expand. Subtract c/a from both sides.

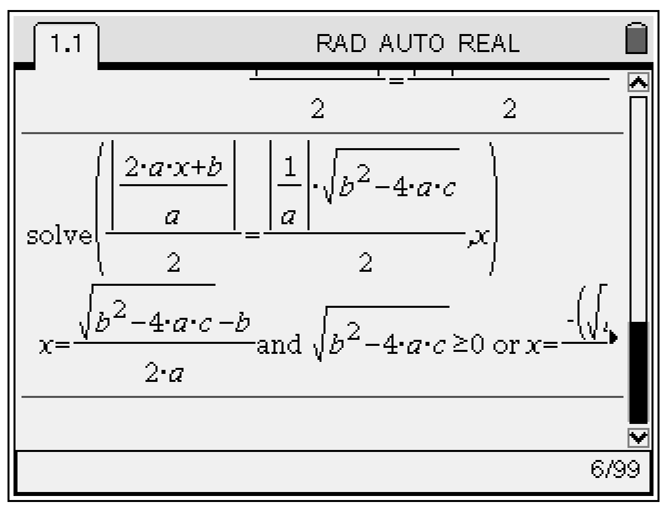
Take half of the coefficient of the x term, square, and add to both sides.





Factor both sides. Take the square root of both sides.





Solve the absolute value equation for x.

Note that you arrive at the familiar quadratic formula.

Computer Algebra Systems Activity: Developing the Quadratic Formula

Student Version:

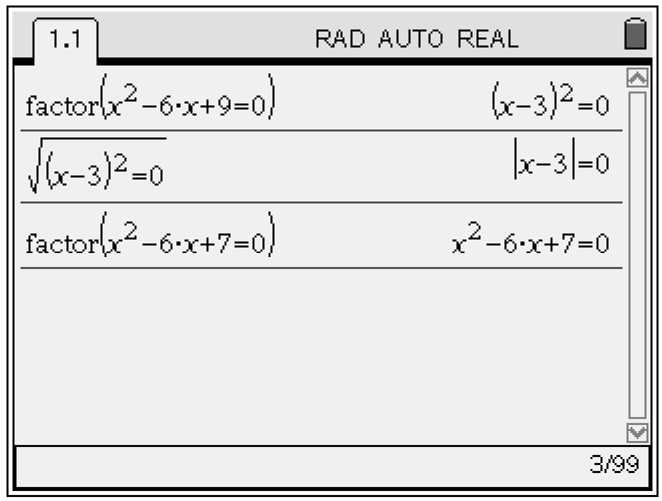
1. Solve the quadratic equation $x^2 - 6x + 9 = 0$.

Press **menu**, select **3:Algebra** and then **2:Factor**.

Type in the equation. Press **enter**.

Take the square root of both sides. **Tip**: You can minimize the typing required by using **ctrl** c and **ctrl** v to cut and paste.

Note that the only solution to the equation $|x - 3| = 0$ is $x = 3$.



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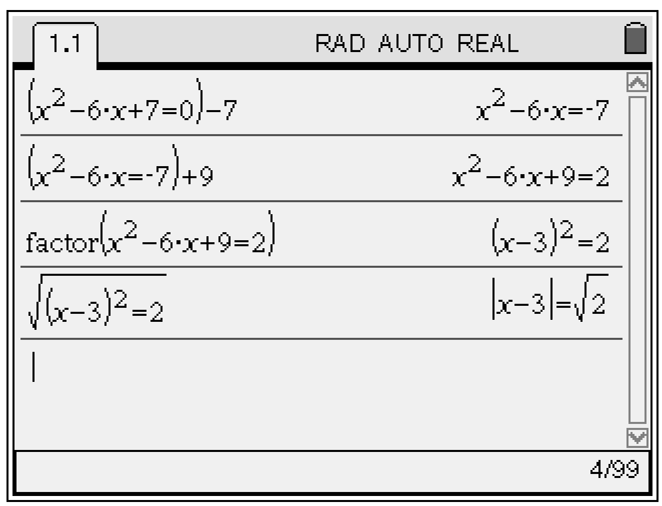

Factor both sides, and then take the square root of both sides, following the pattern from step (1).

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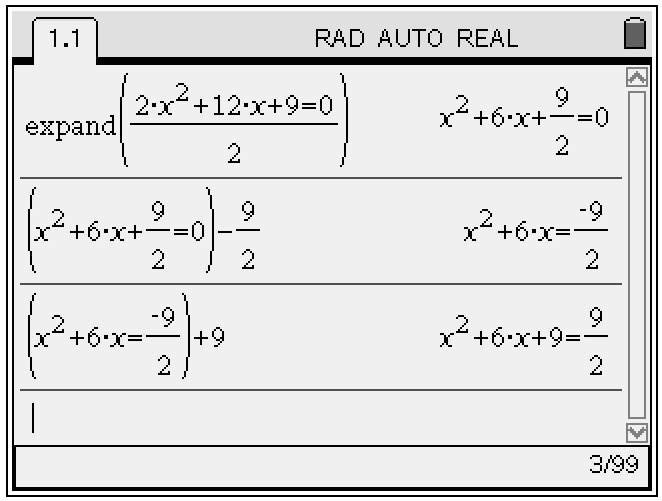

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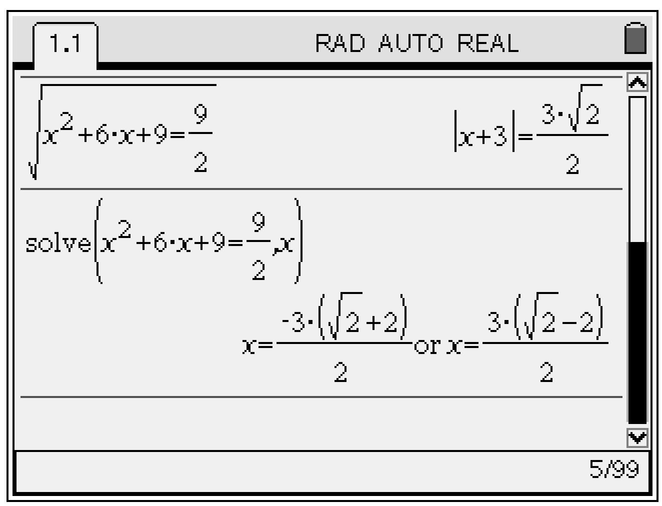

4. You will now use the power of CAS to develop a solution for the general quadratic equation, ax^2 + bx + c = 0. Follow steps similar to those in part (3) to solve for x.

Divide both sides by a, and expand.

Subtract c/a from both sides.

Take half of the coefficient of the x term, square, and add to both sides.

Factor both sides.

Take the square root of both sides.

Solve the absolute value equation for x.

Note that you arrive at the familiar quadratic formula.